OTS: 60-11,323

JPRS: 2283

23 February 1960

30398

CIV-8.0398

DYNAMIC PROBLEMS OF CYBERNETICS

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- USSR -

Distributed by: [redacted]

OFFICE OF TECHNICAL SERVICES
U. S. DEPARTMENT OF COMMERCE
WASHINGTON 25, D. C.
~~Price: $0.75~~

2278

PUBLICATIONS RESEARCH SERVICE
205 EAST 42nd STREET, SUITE 300
NEW YORK 17, N. Y.

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JPRS: 2283

CSO: 3300-N

DYNAMIC PROBLEMS OF CYBERNETICS

/The following is a translation of the introduction and bibliography of the book entitled Dinamicheskiye Zadachi Kibernetiki (Dynamic Problems of Cybernetics) by A. S. Kel'zon, published in Leningrad in 1919; pages 4 to 19, and 287 to 290./

INTRODUCTION

The ten years that have passed since the publication of Wiener's book "Cybernetics" (72) have been years of intensive work by numerous groups of scientists with the aim of creating a general theoretical basis for various branches of sciences that participate in the process of construction of mechanical and electrical systems slated for realizing stable and purposeful operations.

Any machine, ship or airplane may be regarded as a system of mechanical and electrical components that realizes stable and purposeful operations. However, there is an essential difference between the science that studies these machines and cybernetics.

These machines are either intended for the production of routine operations that are not connected with any changes in external conditions, or else these objects (for example, a ship) react to information received from the outside by means of the controling of the action which, in turn, is accomplished by man.

Cybernetics, and in particular that branch of it which is called engineering cybernetics by Tsien, H.S. (71), is a science that integrates information about external conditions, transformation of this information into control signals, and examination and selection of a system of automatic control that realizes particular stable operations.

In classical problems of automatic controk, for example, in the problem of sustaining constant angular velocity of the shaft of a steam engine or in the problem of assuring stability of straight-line uniform flight of an airplane, there seem at first glance to be elements of cybernetics engineering. However, the essential difference of these problems from cybernetic problems is the steady-state of the ideal operation, which must be maintained by the system of automatic control.

Thus, in the problem of regulating the stroke of the steam engine, the ideal operation is shaft rotation with a constant prescribed angular velocity. The system of automatic control is predestined for eliminating or decreasing any deflection of the angular velocity from the prescribed value. Analogously, in the problem of automatic control of airplane motion, the ideal operation is the regular and straight-line motion.

The task of the system of automatic control is to maintain this steady-state operation and reduce to zero all incidental or systematic deviations from this operation.

In the problems of cybernetics, the selection of the ideal operation is an independent scientific problem, and, since the optimum ideal operation of motion is, in the majority of cases, transient in nature, this leads to the development of completely new directions in the science of automatic control.

Thus, the problems of cybernetics include branches which touch upon:

a) the selection and comparative analysis of ideal operations of motion;

b) the selection and comparative analysis of information parameters of external conditons;

c) the selection and comparative analysis of the methods of transformation of information about the external conditions into control signals;

d) the selection and comparative analysis of the system of automatic operation of motion from the point of view of stability and qualitative criteria.

Cybernetics as a science, must develop and systematize the methods of computation, analysis and synthesis applied in engineering practice. The appearance of high-speed alectronic simulating and numerical digital computers led to the tendency to solve concrete problems by means of the methods of approximatenumerical integration of differential equations of motion. However, such an approach does not permit evaluating the problem as a whole and making general conclusions irrespective of the concrete significance of these or other values, as well as the initial conditions of motion.

Thus, cybernetics as a science cannot reject analytical study of a problem.

Analytical study usually becomes possible after some assumptions; the selection of these limitations, made in such a manner that the basic laws of the characteristics are not disturbed, is one of the nicest and most difficult problems of cybernetics.

With this, computing mechanisms retain the greatest significance and are utilized in two directions. From one side, these machines allow fitting analytic investigation to those areas in which it is not possible to obtain a solution in general form. From the other side, after the basic laws have been established, the solution in each concrete case may be refined with the aid of computers by the methods of numerical integration.

Along with the subsequent development of the general state of cybernetics, the application of methods of engineering cybernetics to the solution of concrete problems of contemporary technology has the most important significance.

The application of cybernetics to a number of technological problems was demonstrated in the work of Tsien, H.S. (71): control of the range of ballistic missile, regulation of turboprop engines, automatic control of the operation of turbojet engines, etc. This allowed the author to make a definite contribution also to the development of the general methods of cybernetics.

Thus, the problem of selecting the system of automatic control of the range of ballistic missile allowed Tsien, H.S., to work out, for the given problem, a method of computing the systems of automatic control which are described by means of differential equations in variations with variable coefficients.

Thus, the examination of concrete dynamic problems from the standpoint of cybernetics not only allows discovering the general laws in the given problem, but also to enrichment of the science of cybernetics itself by new, fruitful methods.

In this book, the attempt is being made to examine systematically, from the point of view of cybernetics, one of the problems in the area of automatic control of motion: the problem of missile homing to a moving target.

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At the end of the Second World War, for striking immobile as well as mobile targets (airplanes, enemy ships, etc.), guided reaction-propelled missiles, with a homing head, were utilized for the first time.

The action of such a guided missile in the air or in the water is composed of three stages:

Stage I - approach of the target. During this stage, the reaction-propelled guided missile is brought into the area of the target at a distance within the homing head range. This stage may be achieved by means of a ship, airplane carrier, or missile flight, either free or guided by radio.

Stage II - pursuit of the target. When the distance between the guided missile and the target becomes smaller than the radius of action of the homing head, the outside guidance of the missile is terminated and the missile is guided by the homing head -- a sensitive element whose axis is directed to the target.

Stage III - striking the target. As soon as the distance between the missile and the target becomes smaller than the radius of action of the proximity fuse, the latter is activated and the target is hit by the explosion of the missile. If the missile does not have a proximity fuse, the explosion occurs only at impact. In this case, the second stage must terminate in the accurate hitting of the target by the missile.

The most complex stage, from the point of view of technical realization, and at the same time the most important tactically, is the second stage, insofar as the motion during the second stage is decisive for successful termination of the presented problem -- hitting of the target. The construction of the guided missile, its velocity, the selection of the automatic guidance system realized by means of the homing head, are determined, first of all, by the requirements of Stage II.

In target pursuit of the guided missile with a homing head, various laws of approach may be selected. These laws of target pursuit may be divided into three basic groups.

The first group includes target pursuit along pursuit curves. The basic condition of this motion is that the velocity vector of the center of inertia the missile always passes through the point in space where the target is at the given moment.

The second group includes target pursuit along lead curves. With this, the velocity vector of the center of inertia missile makes a certain angle with the missile-target direction. This angle may be preserved unchanged during motion or may change according to some law.

The third group includes three-point curves or curves of target cover. In this case, the missile, shifting in space, always remains on the radius-vector which passes through the target and some fixed or moving point, for example, starting point. This method of homing is frequently called beam homing.

With a constant lead angle, the trajectory of the missile may become straight if the lead angle is so selected that the velocity vector of the center of inertia the missile is directed into the point of interception. In practice it is very difficult to realize straight-line flight of a missile in view of the complexity of exact determination of the lead angle. This method of homing is called parallel approach, since the line which connects the rocket with the target shifts with this parallel to itself. In this case, the vector of relative velocity of the missile is directed to the target.

The curves of pursuit and three-point curves cannot be straight even with straight-line and constant motion of the target.

In non-rectilinear motion of the center of inertia of the missile, the trajectory curvature is limited by the moment created by the rudder. In connection whit this, the ability of a missile to pursue the target along one of these curves until impact with the target is subject to investigation.

The problem of target pursuit was examined by a number of scientists as a kinematic problem of mutual motion of two points towards mutual approach.

The problem of motion of a point along a curve of pursuit was set for the first time by that man of genius, the Italian artist, scientist, and engineer of the Renaissance, Leonardo da Vinci, in 1510 (51,39).

Independently of Leonardo da Vinci, whose work remained unknown until the end of the Nineteenth Century, this problem attracted the attention of the prominent French scientists Bouguer and Maupertuis in 1732. The first solution of the problem in case of straight-line motion of the target belongs to Bouguer (37). Maupertuis (60) formulated the problem in a more general way, when the target moves with a constant scaler velocity along an arbitrary curve*. The works of Bouguer and Maupertuis in the beginning of the Nineteenth Century became equally forgotten and the problem of the line of pursuit was raised as a new one for the third time by Dubois-Ayme (46). Dubois-Ayme called the pursuit curve the dog's curve. This name has since spread in almost the same way as the original name. The solution of the problem cited in this note turned out to be erroneous, and the honor of discovery of the new problem has been undeservedly attributed to Dubois-Ayme. However, this note has played a positive role, since it induced a series of works which have, to a considerable degree, solved theproblem of the curve of pursuit. The first of these works was the article of Laurent (56). In this thorough investigation, results were obtained anew which are found in Bouguer, and subsequent investigations were conducted; in particular, the curvature radius of the pursuit curve was found, and its limit values at the moment of hitting the target in dependence on correlation of the velocities of both

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*It is necessary to note the inaccuracy allowed in this connection by Loria (58). Loria's reference to the work of Maupertuis:"Solution d'un probleme de geometrie" is incorrect, since this work belongs to Clairaut and has no connection to the curves of pursuit.

points was determined. The problem, as in Bouguer, is limited by the case of the straight line and constant velocity of the target. In subsequent works of Laurent and Sturm (57), the problem is solved for a case unlikely in practice, when the pursuing point participates in transfer motion with a medium which moves progressively with a constant vector velocity. The same problem is elegantly solved by Querret (67). At the same time, he restores the priority of Bouguer and Maupertuis.

Independently and simultaneously with the work of Laurent, an article by Burg was published in Vienna, which was devoted to the problem of the curve of pursuit, raised by Dubois-Ayme. In this work, the problem is solved within a system of oblique coordinates. The surfaces of the solids of revolution are also found, whose generatrix is the curve of pursuit.

Ficklin (48) gave a general formula of the length of the pursuit curve in the case when the pursuing point moves along a straight line and regularly.

The works of Ingalls (54), Barbour (36) and Gosselin (50) are also devoted to various aspects of the problem of Bouguer.

The works of Cesaro (44) are devoted to one particular case of the relative movement of two points, under which the trajectory of one of them is the curve of pursuit.

Nobile (63) applied the methods of natural geometry to the investigation of the curve of pursuit and also developed the ideas of Cesaro.

Gauss (49) discovered certain interesting geometric peculiarities of the curve of pursuit for the case of straight-line movement of the target.

The solution of the problem of Bouguer in the vector form, applying natural axes of coordinates and the formula of Frenet, was given by Puckette (66) in 1953.

Lucas (59) raised for the first time the problem of mutual pursuit along the curve of pursuit of three dogs placed, at the beginning of motion, at the vertexes of an equilateral triangle.

Brocard (38) determined that in this case all three trajectories are logarithmic spirals.

The same problem was solved by Hackett (52) in a more general way by the method of numerical integration. Finally, Morskoy (17) obtained the general condition of the movement of a point along the curve of pursuit, when the latter transforms into a logarithmic spiral.

In 1877, Brocard (40) raised the problem of determining the curve of pursuit in the case when the pursued point moves evenly along a circumference, and the pursuit begins from the center of the same circumference. Having obtained no result, Brocard (40) again raised the same problem in 1383, proposing to limit the solution by setting up the differential equation of the curve of pursuit. This time, the solution was given by Koelhoff (55). This differential equation was brought by Dunoyer (47) to a differential equation of first degree and he also analyzed the integral curves by the method of Poincare. The same problem was solved by the method of graphic integration of differential equations by Morley (61). A qualitative analysis of this problem was given by Hathaway (53) in 1921.

The problem of Maupertuis received further development in the works of Dienger (45) and Morskoy (27). In these articles, the differential equations of the curve of pursuit in the case of an arbitrary flat trajectory of the pursued point are given. Dienger does not examine the possible ways of integrating this differential equation. Morskoy integrates this equation only for three specific cases, from which, in the first case, he returns to the problem examined by Bouguer; in the second case, the curve of pursuit transforms into logarithmic spiral.

D'Ocagne (64) and Burmeister (42) developed a simple geometrical method for plotting the curvature center of the curve of pursuit for the general case of the flat trajectory of the pursued point.

The system of differential equations of the curve of pursuit in the case when the pursued point moves along a space curve was given by Cailler (43) in 1924.

In the work of Pugachev (20), the problems of determining the lead angle in connection with one problem of shooting moving targets is examined.

The dissertation of Bordovsky (21) examines the curve of pursuit for a point of variable mass.

Historical notes of investigations of the curve of pursuit were put together by Archibald and Manning (35); they are also cited in the works of Loria (58), Neville (62), and Puckette (66).

The kinematic problem of movement along a three-point curve was examined for the first time by Wilder (68). Having put together the differential equation of the curve, Wilder determined the curvature at any point of the trajectory, asymptote, inflection point, and other elements of the curve. Further, he constructed all of the solutions without integrating the differential equation of the curve. At the end of his work, Wilder points to possible ways of integrating the differential equation, refusing to apply them himself due to their complexity.

During and after the Second World War, works appeared which were devoted to the movement of homing missiles which pursue a target.

In the work of Perret, Roth, Sänger, and Vaellmy (69), the problem of pursuing a target which moves horizontally, equally, and along a straight line, by a rocket which is guided by a gas jet is examined. The flight occurs along a three-point curve. The differential equations of the motion are integrated by the numerical method for a number of initial conditions. The doctoral dissertation of Roth (73) is devoted to the same problem. In both works, the authors limit themselves by the determination of the trajectory of the rocket and oscillations of the body at the gravitational center.

In a work by the German guided-missiles specialist Leisegang (82), a kinematic investigation of the movement of a rocket guided by the method of target coverage is conducted. In an investigation by means of this method, the point from which the shot is fired, the missile, and the target are always on the same straight line. The rocket describes a trajectory which is called the three-point curve. Leisegang regards the missile and the target as points which move with constant velocities. The trajectory of the target is the straight line.

The author analyzes a case of shooting from ground to an airplane and finds the differential equation of the trajectory of the missile guided by the method of target coverage. The obtained differential equation may be reduced to quadratures which are expressed in elliptical functions. Therefore, the author selects for further investigation another method -- the method of approximated structure of trajectories of the guided missiles. The obtained differential equation of the three-point curve is utilized by the author for finding the formula of the radius of the trajectory curvature of the guided missile. Having obtained the value of the curvature radius, Leisegang determines the locus of the point of maximum curvature or minimum radii of curvature.

A subsequent investigation is conducted by means of constructing approximated, three-point trajectories of the missile, depending on various angles of starting, on various distances of the trajectory to the target, as well as on various correlations of velocities of the missile and the target. Each investigation is conducted separately; furthermore, in a study of the various starting angles, the distance to target and correlation of the velocities of the missile and target are taken to be constant. In the same manner, the influence of two other factors is investigated: the change of distance to target and the correlation of missile and target velocities.

Further on in the work, three-point trajectories obtained in various cases of shooting from one airplane to another are discussed. The method of their approximated construction does not differ in principle from the construction of curves in shooting from ground to airplane, only in this case new independent factors are added: the speed and the direction of flight of the attacking airplane. Making up his mind as to the value of the smallest possible radius of curvature, the author gives a method for calculating errors of hitting in unfavorable correlations of the speeds of the missile and the target, when the curve of the trajectory at any point becomes so large that the missile cannot follow further along the three-point trajectory. It is proposed with this that the missile, beginning at this critical moment, moves along an arc of the circumference of the smallest tolerable radius of curvature.

Two works of Rössler (83), the prominent German specialist in remote control, are devoted to target pursuit along the curve of pursuit and with a constant angle of advance. Regarding the missile and the target as loci which move with constant, scalar velocities at the time when the target moves along a straight line, Rössler finds the equations of the trajectories of the missile, also the curve of pursuit in constant angle of advance.

Utilizing the equations of the trajectories, the author determines for both cases the law of change of the radius of curvature of the curves and the condition under which the distance between the rocket and the target decreases monotonously. This part of the work has the most important theoretical significance.

The subsequent analysis of Rössler is reduced to a calculation of the probability of hitting the target. The author makes up his mind as to the minimal tolerable radius of curvature of the trajectory of the missile proceeding from the fact that the acceleration which the missile may achieve is limited by its rudder and mass characteristics. The target is considered as having been hit not only in a mathematically precise hit, but also in the case when the distance between the missile and the target becomes smaller than the radius of action of the proximity fuse.

Rössler feels that, after reaching the maximum tolerable acceleration (the smallest radius of curvature), the rudders remain in the same extreme position and the missile, having departed from the curve of pursuit or the curve of the constant angle of advance, describes an arc of a circumference which is located in the same plane as the curve of pursuit. If, after such a flight, the axis of the homing head will again find itself directed to the target, then, from this moment on, the missile will begin to move along the second curve of pursuit.

The probability of hitting the target is determined by Rössler in various limits of equiprobable values of the initial angle of attack: a) when the pursuit may be commenced from any direction, and, b) when the pursuit commences from behind, from the sector limited by 60° angles from the direction precisely into the tail. For the curve of pursuit, Rössler determines how the probability of hitting changes under various correlations of velocities of the missile and the target, as well as under the change of only the speed of the target.

For a flight along a curve of the constant angle of advance, Rössler determines the change of the probability of the hit if the axis of the homing head makes up a constant angle with the direction of the tangent to the missile trajectory.

Thus, in the works of Rössler, as well as in the works of Leisegang, the authors confine themselves to the investigation of the trajectories of the curves of pursuit: the curve of pursuit, the curve of the constant angle of advance, and the three-point curve.

The content of these works is the kinematic problem of the motion of two points of which one moves along a straight line and constant velocity and the other pursues the first with a constant speed along these curves. With this, it is necessary to note that even this simplest kinematic problem has not been solved to the end: the equations of the movement of the point have not been obtained in any of these works.

The attempt to relate the obtained kinematic data (the equations of the trajectories) with the dynamics of the missile, while making up one's mind about the minimum radius of the curvature of trajectory which can be achieved by the center of inertia of the rocket, is a first and extremely inaccurate approximation in this direction.

Together with this, these works allow drawing certain general conclusions about the tactics of utilization of guided missiles with homing heads, irrespective of the construction and dynamic characteristics of the specific missiles.

Beginning with 1945, many works appear in print in the United States of America devoted to the problem of a homing system of guidance. One of the first is the work of Newell (78), in which the kinematic problem of the relative motion of two points is analyzed, of which one - the target - moves along a straight line and uniformly, and the other - the pursuing point - moves according to one of five methods of guidance and homing: a) guidance by a beam; b) homing with zero angle of advance (along the curve of pursuit); c) homing with a constant angle of advance; d) homing by the method of parallel approach; e) homing by the method of proportional navigation.

In the next year, 1946, the work of Spitz (79) appears, in which the method of proportional navigation is examined as a kinematic problem of mutual motion of two geometric points: target and missile.

In the works of Yuan (74), Bennet and Mathews (75), various methods of homing (the method of proportional navigation, the curve of pursuit, parallel approach) are examined; furthermore, analytic and approximate numerical methods of solution of the kinematic problem of relative motion of two geometric points are utilized.

In the work of Adler (76), the method of proportional navigation is examined in pursuit of the target in three-dimensional space, when previously, except for the work of Cailler (43), all investigations were limited by the study of the plane problem. Adler also limits himself by the examination of the kinematic problem of the missile motion - the locus which pursues the target.

In the book of Locke (70), a summary of American research in the area of guidance, performed during the ten postwar years, is given.

On one hand, in this work, the trajectories of a missile which is regarded as a point which moves along the curve of pursuit with a constant angle of advance, are analyzed according to the method of parallel approach, according to the method of proportional navigation, and in guidance by the beam. These kinematic investigations are connected with the dynamics of the missile by means of an examination of the character of the change of normal acceleration or angle velocity of turn of the tangent near to the target approaches zero, then this is sufficient to assure the accurate flight of the missile to the target. At the same time, Locke states that the angle of turn of the rudder is proportional to the angular velocity of the turn of the tangent.

Examining, on the other hand, the dynamics of the missile during guidance of the rudder along one of the five selected methods of pursuit and applying to the differential equations of missile motion the law of the rudder turning as functions of the angle of error, Locke, entering into a contradiction with his previous conclusions, states (on the basis of simulating on electronic computers) the inevitability of a miss and the absolute necessity of shutting off the homing system near the target. This incorrect evaluation of the whole problem of homing is prevalent at the present time. Thus, Adler (76), examining the method of proportional navigation, states analogously: "It is possible to demonstrate that for $z_{(p)} = 1$ (i.e., if the missile is taken as a material point) this equation always leads to the missile hitting the target (for point objects); furthermore, this occurs during the final time interval and for a finite value of the missile acceleration. This conclusion does not change even in the presence of errors of aiming or manoeuvers of the target. For a real missile, which has a transfer function that differs from one, the points that represent the missile and the target will never coincide".

This contradiction, which was called by Baxter (30) "the mathematical paradox of the curve of pursuit" does not find its needed explanation in the works of Locke and Adler.

Locke (70) utilizes the kinematic investigation of the relative motion of the missile for selection of the system of automatic control of the motion. For this, on one hand, the amplitude-frequency spectrum of the angular velocity of the turn of the tangent to the trajectory of the missile, the value is constructed that is proportional to normal acceleration (since the scalar velocity is constant). On the other hand, the amplitude-frequency spectrum is constructed for the selected system of automatic control of motion of the missile. Comparison of these two amplitude-frequency spectrum serves as a theoretical criterion of the quality of the selected system of automatic control. This method is based on the assumption that the angle of turn of the rudder is proportional to the angular velocity of the turn of the tangent -- which is taken by Locke (70) and by other researchers as indisputable and as requiring no proofs.

In fact, as will be demonstrated below, this assumption is incorrect; it does not take into account the dynamics of the missile itself as a solid body, it leads to considerable errors, and in a number of important cases, it leads to results which are simply contrary to the truth.

Independently of motion along ideal trajectories, Locke (70) regards the real motion of the guided missile. The differential equations of the motion of the missile are numerically integrated together with the selected law of rudder control. The law of rudder control is selected depending on which of the five ideal trajectories the missile should move near. With this, it is natural that the real trajectories, even in the absence of perturbances in the motion of the target, as well as in the motion of the missile, will differ from the ideal ones, since the law of rudder control along the angle of error may assure close, but not precise motion, along the ideal trajectory.

The appearance of contemporary high-speed electronic computers led to a tendency to compute these near, real, perturbed trajectories separately and, proceeding from this numerical integration of differential equations of motion, to select the coefficients of the system of automatic control. Such an approach to the problem is especially attractive, since the simulation devices allow numerically integrating linear of material points, and a strict proof of the necessary and sufficient conditions when an investigation of stability, according to the first approximation with application of linearized equations, is in order. In the same work, Lyapunov proposed and developed his "second method", which is a powerful means of determining the stability of motion of any non-linear systems.

Lyapunov's second method, applicable to the solving of non-linear problems of the theory of automatic regulation, found further fruitful development in a series of important works by Lurie which were collected by him in a monograph (12).

The ideas of Lyapunov received further essential development in works by Chetayev (27) and Malkin (14). For solving analogous problems, the theorems of Lyapunov were applied in works by Letov (9). The non-linear problem of auto-oscillations of a stand with automatic pilot was examined by Butenin (2).

The linear theory of motion stability and automatic regulation received a powerful method of investigation in operations analysis. The methods of operations analysis applicable to problems of mechanics are worked out in a monograph of Lurie (11), which played an important role in the expansion of the methods of operations analysis in engineering computations.

Interesting works in the area of linear systems of automatic regulation are given by Tsypkin and Bromberg (26), Krasovsky and Pospelov (6), (7).

Beginning with the works of Vyshnegradsky (5), the transitional processes were investigated after termination of the action of perturbing forces. Lately, in the works of Bulgakov (1), Malkin (14), and Moiseyev (16), methods of analyzing the stability of motion in constant (but limited, according to value) external perturbing forces are being worked out. In these works, ideas are being developed which were indicated by Lyapunov (13). Methods of analysis of non-linear forced oscillations are also analyzed in the works of Butenin (3).

The important work of Neymark (18) allowed generalizing and determining the general point of view to different criteria of the stability of motion.

In the area of simulating schemes of automatic regulation, the works of Trapeznikov and Kogan (24) are essential.

The investigation of the dynamics of guided missiles, whose force of thrust is of reactive origin, is based on the doctrine of the dynamics of a point and a solid body of variable mass. The originator of this doctrine was Meshchersky (15).

differential equations with variable coefficients, as well as non-linear differential equations.

However, in such a napproach to the selection of the basic coefficients of the system of automatic control, it is not possible to say anything of the stability of motion in the sense of Lyapunov, as well as in the sense of technical stability - stability at the final time interval.

As is correctly pointed out by Tsien (71), it is necessary for examination of the problem of stability of motion to set up and examine the differential equations of motion in variations at the time when, in simulation, the differential equations of perturbed motion are being integrated. The equations in variations cannot be obtained without the solution of the dynamic problem of ideal motion of the missile.

Aside from this, the numerical integration of differential equations of perturbed motion, which is done by electronic simulating devices, does not allow determining the general laws of the ideal as well as of the real perturbed motion, since each solution depends essentially on the initial conditions of motion and the concrete values of the constant parameters which participate in the differential equations of motion.

Thus, for example, in approximated numerical integration of the differential equations of perturbed motion, it is not possible to separate those components of a miss that are induced by the inability to realize to the end the ideal aim according to the selected method (due to dynamic limitations of the system) from those components of the miss that were induced by the difference of the law of rudder control in real motion from the law of turbing the rudder in ideal motion, and conditions of stable aim of the missile at the target connected with this.

Thus, for the further development of the art of homing, it is necessary to solve the dynamic problem of the ideal motion of the missile which, on one hand, would allow determining the basic characteristic laws of the process and the dependence of the accuracy of the hit on the dynamic and kinematic parameters of the missile and the target. On the other hand, the solution of this problem will allow obtaining a system of differential equations in variations, which is the basis for selecting the scheme of automatic control and determining the conditions of stable aiming of a rocket at its target.

The theoretical basis for solving the problem of the stability of rocket motion is the fundamental work of Lyapunov (13). In this work, Lyapunov gave a mathematically accurate definition of the stability of the motion of a system

These ideas received further development in the works of Kosmodem'yansky (8).

The problem of motion of controlled missiles is beyond the scope of external ballistics. This problem, due to its set-up and methods of solution, is one of the classic problems of cybernetics. all of the basic features and methods of this new science find their place in the investigation of the problem of homing.

Comparative analysis of the ideal systems of motion; selection of the optimum parameters that transmit the information regarding external conditions; rational methods of transforming information about external conditions into the control signals, and, finally, synthesis of a system of automatic control of motion -- such are the most important composite parts of the doctrine of homing.

1. Setting-Up the Problem and Methods of Its Solution.

A critical Survey of the methods of analysis and synthesis of homing systems allows planning a further path of investigation. The first problem is the analytical study of the dynamics of ideal motion of a missile. This division logically develops the kinematic study of various methods of homing. The kinematic investigation (70), (78), (79) was conducted under the following limitations:

a) the missile and the target were regarded as loci;

b) the motion of the target was taken to be regular and along a straight line;

c) the scalar velocity of the missile was constant.

Under these conditions, the kinematic equations of motion were integrated and the following were determined: the trajectory of motion; normal acceleration; the angular velocity of the turn of the tangent; the time of the process of target pursuit; the threshhold values of normal acceleration and the angular velocity of the turn of the tangent to the trajectory near the target.

The advantages and disadvantages of the various methods of homing were judged by the character of the changes of normal acceleration or of angular velocity of the turn of the tangent to the trajectory, which are equivalent for a constant value of the velocity.

Such an evaluation of the possibility of realization of the pursuit up to striking the target is analogous to the one applied widely in aviation; namely, according to the value of overload, or according to the radius of turn assumed in the dynamics of a ship.

These evaluations give good results in established, steady-state motions. However, as the analysis shows, in a non-steady-state process of homing this evaluation leads to considerable errors, and, in certain areas of values of parameters, leads to results directly opposed to reality.

The paradoc of the curve of pursuit is contained in the fact that the normal acceleration near the target may approach zero, the radius of the curvature of the trajectory may increase without limit, the overload may approach unity, i.e., be absent, and the missile will not be able to realize the motion along the given, ideal trajectory, since the angle of turn of the rudder, necessary for assurance of the ideal motion near the target, must increase without limit.

Analytical study of the dynamics of the ideal motion of a rocket is conducted under the following limitations:

a) the motion of the target -- uniform and along a straight line;

b) the scalar velocity of the missile is constant.

Unlike the case of kinematic investigation, the rocket is regarded as a solid body with a rudder, which moves under the action of the force of thrust, aerodynamic forces, and moments, as well as the force of gravity. Solving the dynamic problem of ideal motion of a missile allows judging the possibility of realizing the pursuit along the selected trajectory on the basis of the law of change of angle of the turn of rudder, while taking into account the inertia of the missile itself and the action of forces and moments.

Comparing the critical behavior of the angle of turn of the rudder and the normal acceleration of the center of inertia of the missile near the target allows us to solve the "mathematical paradox of the curve of pursuit", (Bakster, /80/).

The dynamic analysis ofthe ideal motion allows judging whether, under the given initial conditions of motion, the correlation of velocities of the missile and the target, as well as in dependence on other kinematic (the angle of advance) and dynamic (the moment of inertia, the forces and moments which influence the rocket) characteristics, the guided missile can pursue the target until a mathematically-accurate hit or whether it will be able to determine the value of the miss, if the rudder, whose angle of turn is limited by braces, will arrive at the extreme position before the process of target pursuit will be determined. In the latter case, the formula for the angle of turn of the rudder in combination with the equation of the curve of pursuit serves for determining the value of the miss.

Thus, solving the problem of the motion of a guided missile along the ideal curves of pursuit, it is possible to obtain a result about the dynamic possibilities of the missile, about the influence of various design factors, correlations of the velocities of the missile and the target, the angle of advance and initial conditions of motion to the possibility of striking the target.

A study of the dynamics of the ideal motion of a missile under various methods of homing: 1) with a zero angle of advance, when the velocity vector of the rocket is uninterruptedly directed to the moving target; 2) with a constant angle of advance, when the velocity vector of the missile makes a constant angle with the line which connects the missile with the target (the target line, the guide line or distance vector); 3) with parallel approach, when the line of the target remains parallel to itself and the vector of relative speed is uninterruptedly directed to the target; 4) proportional navigation, when the angular velocity of the turn of the tangent to the trajectory of the missile is proportional to the angular velocity of rotation of the target line, -- allows comparing these methods and selecting from them those that assure the smallest or zero miss.

Comparative analysis of various methods of homing is conducted on the basis of the law of rudder turn, which corresponds to each of the methods of homing. At the same time, this analysis allows answering the question regarding the selection of the most rational parameters which give the information about external conditions -- in the given case, about the values which characterize the motion of the target.

The second problem is the analytical study of the real motion of the missile which is realized by means of a scheme of automatic control which seeks to return the missile to one of the ideal homing trajectories.

The curves which are close to ideal and which are realized by the missile under automatic control of motion by means of homing heads, are called the real curves of pursuit. The homing head, whose axis is constantly directed to the target, leads the missile by means of a system of automatic control, giving a command to the rudder that is constantly directed to elimination of the angle of error.

In this case, the system of differential equations of perturbed or real motion is determined by the law of rudder turn as a function of the angle of error.

Utilizing the closed form of the obtained solution of the problem of ideal motion of the missile and the differential equations of the real motion, the differential equations of motion in variations are found. If the law of rudder control is built up directly as a function of measured error in angular motion, the system of differential equations in variations has essentially variable coefficients. The selection of parameters of increase of the law of control in the case of a system of differential equations in variations with variable coefficients has not been developed in a general form. For some particular problems, in this case, the method of adjoint functions of Bliss (71) [sic] is applied; however, for solving the problem of the stability of the missile in the process of homing, this method cannot be utilized. Therefore, the investigation of the stability of motion and the selection of the basic scheme of automatic control of motion in homing, is based on transformation of the parameters of of error, measured during motion, into control signals which permit obtaining a system of differential equations in variations with constant coefficients for flight in a horizontal plane (or in any plane, if the component gravitation force is not taken into account). Constant transformation of the measured parameter of error into a controlling signal is possible due to the previously obtained solution of the problem of dynamics of ideal motion of the missile, and is performed by means of a computing device.

For the system of differential equations in variations with constant coefficients, the selection of increase parameters in the law of control of rudder is solved by classical methods. The system of automatic control of motion may be made self-adjustable or multi-accomodating, in the terminology of Tsien (71); furthermore, the principle of self-adjusting is based on the solution of the problem of dynamics of the ideal motion of the missile. The solution of the problem of dynamics of the real motion of the missile allows judging deviations from the ideal trajectory, which are induced, first of all, by the difference of the law of rudder control in real motion from the law of turning of the rudder in ideal motion and, in the second place, by initial perturbances.

The analysis of the influence of the gravitational force on the stability of motion in the case when the plane of pursuit does not coincide with the horizontal plane, is an important area in the study of the process of homing for ideal, as well as for real, motion.

Subsequent refinement of the solution: taking into account the change of scalar velocity of the missile, variability of mass, change of atmospheric density, etc... may be realized by means of numerical integration on simulating devices for concrete initial conditions of motion and particular values of all parameters which enter into the differentia equations of motion. Such a subsequent checking of the selected construction and sheme of automatic control is necessary, but it is quite evident that in no measure can it replace analytic analysis of the problem.

Thus, the analysis and synthesis of the process of homing falls under three heads:

a) ideal motion of the missile, -- which allows determining the dynamics of the missile without dependence on the system of automatic control of motion in a general way, but with taking into account the selected law of homing, the initial conditions of motion, and all other dynamic and kinematic characteristics of the missile and the target;

b) real motion of the missile, -- for which, utilizing the obtained solution of the problem for ideal motion, the system of differential equations in variations is examined, which allows determining the values of coefficients of the system of automatic control, which assures the stable aim of the rocket at the target;

c) determining the way to transform measured parameters of error and controlling signals and setting up conditions under which the homing system may be made self-adjustable.

The analysis of the process of homing is equally applicable to missiles which move in the air and in the water.

Basic Symbols

a - distance between the missile and the target.

x, y - coordinates of the missile

xs, ys - coordinates of the target

v - velocity of the rocket

v_s - velocity of the target

ψ - angle of turn of the line of target under zero and constant angles of advance

η - the angle of turn of the line of target under proportional navigation

α - angle of attack, angle of glide

φ - the angle of turn of the tangent to the trajectory of the missile under proportional navigation.

β - angle of turn of the rudder

$k=\frac{vs}{v}$ - the ratio of velocities of target and missile.

Bibliography

a) Domestic

1. Bulgakov, B. V. On the Problem of Forced Oscillations of Pseudo-Linear Systems. P. M. and M, VII, Issue 1, 1943.

2. Butenin, N. V. On the Theory of Resonance in the Mechanical Auto-Oscillating System with Gyroscopic Members, P.M. and M, Issue 1, 1950.

3. Butenin, N. V. On the Theory of Forced Oscillations, P.M. and M, Issue 4, 1949.

4. Vinogradov, I. M. The Principle of the Theory of Numbers. Gostekhizdat, 1952.

5. Vyshnegradskiy, I. A. a) On the General Theory of Regulators. Petersburg, 1876; b) On the Regulators of Direct Action. Petersburg, 1877.

6. Krasovskiy, A. A. On the Degree of Stability of Linear Systems. Moscow, 1948.

7. Krasovskiy, A. A. and Pospelov, G. S. Some Methods of Computation of Approximated Time Characteristics of Linear Systems of Automatic Contro. A and T, V. XIV, Issue 6, 1953.

8. Kosmodem'yanskiy, A. A. The Mechanics of Bodies of Variable Mass, 1947.

9. Letov, A. M. Stability of Non-Linear Controlled Systems. Gostekhizdat, 1955.

10. Loytsyanskiy, L. G., and Lur'ye, A. I. The Course of Theoretical Mechanics, II. Gostekhizdat, 1948.

11. Lur'ye, A. I. Operational Computation and Its Applications to the Problems of Mechanics. Gostekhizdat, 1950.

12. Lur'ye, A. I. Some Non-Linear Problems of the Theory of Automatic Control. Gostekhizdat, 1951.

13. Lyapunov, A. M. The General Problem of Stability of Motion. Khar'kov, 1892.

14. Malkin, I. G. The Theory of Motion Stability. Gostekhizdat, 1952.

15. Meshcherskiy, I. V. The Dynamics of a Point of Variable Mass. Petersburg, 1897.

16. Moiseyev, N. D. The Quasi-Integral Derivation of Coefficient Criterion of Stability for the Simple System of Non-Uniform Differential Equations, Moscow, 1948.

17. Morskoy, I. V. The Curve of Pursuit, The News of North-Caucasian Industrial Institute in Novocherkassk, Vol (15), 1935.

18. Neymark, Yu. I. The Structure of D-Division of the Space of Polynomials and Diagram of Vyshnegradsky and Nyquist, DAN, v. IX, No. 5, 1948.

19. Privalov, I. I. Introduction to the Theory of Functions of a Complex Variable. Gostekhizdat, 1948.

20. Pugachev, V. S. The Generalization of the Problem of the Curve of Pursuit, P.M. and M, v.X, 1946.

21. Borodovskiy, P. V. On the Problem of the Line of Pursuit for the Point of Constant and Variable Mass. Author-Abstract of Candidate's Dissertation, Odessa, 1956.

22. Ryzhik, I. M., and Gradshteyn, I. S. Table of Integrals, Series, Sums and Products. Gostekhizdat, 1951.

23. Smirnov, V. I. Course of Higher Mathematics, Gostekhizdat 1951.

24. Trapeznikov, V. A., and Kogan, B. Ya. The Principle of Construction of Simulating Devices for Analysis of Processes of Automatic Control. A and T, v. XIII, issue 6, 1952.

25. Fikhtengol'ts, G. M. Course of Differential and Integral Computation, Gostekhizdat, 1948-1949.

26. Tsypkin, Ya. Z., and Bromberg, P. V. On the Degree of Stability of Linear Systems. News of AN SSSR, OTN, No. 12, 1945.

27. Chetayev, N. G.
   a) The Stability of Motion, Gostekhizdat, 1946.
   b) On Stable Trajectories of Dynamics. Scientific Notes of Kazan University, Book 4, Issue 1, 1936.
   c) On Theorem of Non-Stability, DAN, VI, No. 9, 1934.
   d) The Theorem of Non-Stability for Proper Systems, PM and M, Vol. XII, Issue 5, 1944.
   e) On Some Problems of Stability and Non-Stability for Improper Systems, PM and M, Vol. XII, Issue 5, 1948.

Accepted abbreviations: A and T - the Journal "Automatics and Telemechanics"; PM and M - the Journal "Applied Mathematics and Mechanics"; AN SSSR, OTN - Academy of Sciences USSR, Division of Technical Sciences; DAN - Reports of Academy of Sciences USSR.

b) Literature in Translation

28. Ayns, E. L. Simple Differential Equations. Khar'kov, 1939.

29. Gursa, E. A Course of Mathematical Analysis, v.I-III, 1933-1936.

30. Kamke, E. The Manual of Simple Differential Equations, Moscow, 1951.

31. Muller, F. Telecontrol, Publ. IL, Moscow, 1957.

32. Piadzhio, G. The Integration of Differential Equations, Gostekhizdat, 1933.

33. Rosser, D., Newton, R., and Gross, G. The Mathematical Theory of the Flight of Unguided Rockets, Publ. IL, Moscow, 1950.

34. Satton, D. Rocket Engines, Moscow, 1952.

c) Foreign Literature

35. Archibald, R. C., and Manning, H. P. Remarks and Historical Notes. The American Mathematical Monthly, XXVIII, 1921, p. 91-92.

36. Barbour, L. G. Curve of Pursuit Generalized. Analyst (Des Moines), 1879. P. 108-122.

37. Bouguer, P. Sur de nouvelles courbes, auxquelles on peut donner le nom de lignes de poursuite. Memoire de mathematique et de physique de l'Academie royale des sciences. Paris, 1732.

38. Brocard, H. Questions No 251. Nouvelle Correspondence mathematique, III, 1877, P. 280.

39. Brocard, H. Notes sur divers articles de la nouvelle correspondence. Nouvelle correspondence Mathematique, 1880, p. 211-213.

40. Brocard, H.
a) Questions proposees No. 250, Nouvelle correspondence mathematique, III, 1877, P. 175.
b) Mathesis, III, 1883, P. 232.

41. Burg, A. Untersuchungen über eine besondere krumme Linie. Jahrbuch des Polytechnischen Institutes in Wien;, B. 4, 1823, S. 508-531.

42. Burmester, L. Lehrbuch des Kinematik. Erster Band. Leipzig, 1888, S. 63.

43. Cailler, Ch. Introduction Geometrique a la mecanique rationelle. Geneve et Paris, 1924, P. 508.

44. Cesaro, E. a) Proprietes d'une courbe de poursuite. Nouvelles annales de mathematiques, 3 Serie, II, Paris, 1883.

45. Dienger, I. Fragen aus der Mechanik. Archiv der Mathematik und Physik mit besonderer Rücksicht auf die Bedürfnisse der Lehrer an höheren Unterrichtsanstalten, B. 15, Greifswald, 1850, S. 335-340.

46. Dubois-Ayme, De la courbe que decrit un chien en courant apres son maitre. Correspondence sur l'ecole imperiale polytechnique. Tome second, No. 3, Paris, 1811, P. 275.

47. Dunoyer, L. Sur les courbes de poursuite d'un cercle. Nouvelles annales de mathematiques, 4 Serie, VI, Paris, 1906.

48. Ficklin, J. To Find a General Formula for the Length of Curves of Pursuit. Proceedings of the American Association for the Advancement of Sciences, V. 20, 1871, P. 63-64.

49. Gauss, A. F. Über Kurven, welche die Eigenschaft haben, dass je zwei Tangenten aus einer gegebenen Geraden eine Strecke ausschneiden, welche zu dem von den Berührungspunkten Begrenzten Bogen in einen gegebenen Verhältnisse steht. Progr. Bunzlau, 1890.

50. Gosselin. Patte de chien probleme. Memoires de l'Academie des Sciences de Metz, 1858-1859, P. 459-468.

51. Günther, S. Studien zur Geschichte der mathematischen und physikalischen Geographie. Halle a/s 1879, S. 42-45.

52. Hackett, F. E. A Numerical Solution of the Triangular Problem of Pursuit. The Johns Hopkins University Circular, No. 7, Baltimore, 1908, P. 135.

53. Hathaway, A. S. Solution of Problem 2801. The American Mathematical Monthly, XXVIII, 1921, P. 93-97.

54. Ingalls, J. M. Curves of Pursuit. Analyst (Des Moines), VII, 1880, P. 89-96, 117-118.

55. Koelhoff. Solution de question 291. Mathesis, VI, 1886, P. 135.

56. Laurent, Th. de St., Solution du probleme des courbes de poursuite. Annales de mathematiques pures et appliques, XIII, Paris, 1822-1823.

57. Laurent, Th. de St., et Sturm, Ch. Extension du probleme des courbes de poursuite. Annales de mathematiques pures et appliquees, XIII, Paris, 1822-1823.

58. Loria, G. Spezielle algebraische und transzedente ebene Kurven. Theorie und Geschichte. Zweiter Band. Leipzig une Berlin, 1911, S. 241.

59. Lucas, E. Questions proposees No. 251. Nouvelle Correspondance mathematique, III, 1877, P. 175-176.

60. Maupertuis, P. L. M. Sur les courbes de poursuite. Memoire de mathematique et de physique de l'Academie royale des sciences. Paris, 1732.

61. Morley, F. V. A Curve of Pursuit. The American Mathematical Monthly. XXVIII, No. 2, 1921, P. 54-61.

62. Neville, E. H. Mathematical Notes, No. 1004. The Mathematical Gazette, XV, No. 214, London, 1931.

63. Nobile, V.
a) Sullo studio intrinseco delle curve di caccia. Rendiconti del Circolo matematico di Palermo, XX, 1905.
b) Sul problema delle curve di caccia. Giornale di matematiche, Napoli, XLVI, 1908.

64. Ocagne, M. d'. Sur le centre de courbure des courbes de poursuite. Bulletin de la societe mathematique de France, XI, No. 3, 1883, P. 134.

65. Pearson, K. Tables of the Incomplete Betafunction. Cambridge, Biometrika, 1934.

66. Puckette, C. C. The Curve of Pursuit. The Mathematical Gazette, 37, No. 322, London, 1953, pp. 256-260.

67. Querret, J. J. Developpements et note historique sur les memes courbes. Annales de mathematiques pures et appliques. XIII, Paris, 1882-1883.

68. Wilder, Ch. F. A Discussion of a Differential Equation. The American Mathematical Monthly, XXXVIII, No. 1, 1931.

69. Perret, E., Roth, E., Sänger, R., Vaellmy, H. Flugbahnen von Leitstrahlraketen mit Gasstrahlsteuerung. Zeitschrift für angewandte Mathematik und Physik, III, No. 4, Basel, 1952, S. 241-258.

70. Locke, A. Guidance. New York, 1955. (1957).

71. Tsien, H. S. Engineering Cybernetics. New York, 1954.

72. Wiener, N. Cybernetics or Control and Communication in the Animal and Machine. New York, 1948.

73. Roth, E. Zur Berechnung der Flugbahnen von Leitstrahlraketen. Zürich, 1953.

74. Yuan, L. C.-L. Homing and Navigational Courses of Automatic Target-Seeking Devices. Journal of Applied Physics, vol. 19, No. 12, 1948, p. 1122.

75. Bennett, R. R., Mathews, W. E. Analytical Determination of Miss Distances for Linear Homing Navigation Systems. Hughes Aircraft Company Technical Memorandum, No. 260. 1952.

76. Adler, F. P. Missile Guidance by Three-Dimensional Proportional Navigation, Journal of Applied Physics, Vol. 27, No. 5, 1936, pp. 500-507.

77. Billington, J., Cole, A., and Lamb, B. Defence Against the ICBM, Aeroplane, No 2356, pp. 629-632, No. 2357, pp. 662-665, 1956.

78, Newell, H. E., Jr., Guided Missile Kinematics. Naval Research Laboratory of USA, 1945.

79. Spitz, H. Partial Navigation Courses for a Guided Missile Attacking a Constant Velocity Target. Naval Research Laboratory of USA, 1946.

80. Baxter, J. P. Scientists Against Time. Boston, 1946.

81. Rosenberg, R. M. A Pursuit Problem. Journal of the Franklin Institute, vol. 262, No. 4, pp. 265-279.

82. Leisegang, H. Die Bahnkurven der nach Deckungsverfahren ferngelenkten Geschossen. Deutsche Luftfahrtforschung, 1944.

83. Rössler, Die Zielvolgerung nach der Verfolgungskurve und Vorhaltskurve. Deutsche Luftfahrtforschung, 1944.

84. Grigor'yeva, O. V., and Kel'son, A. S. The Dynamics of Proportional Navigation. Scientific Notes of Leningrad Higher Engineering Naval Academy Admiral im. S.O. Makarova, Issue 12, 1958.

85. Kel'zon, A. S. The Equation of Motion of a Point Along the Curve of Pursuit. Scientific Notes of the Leningrad Higher Engineering Naval Academy imeni admiral S.O. Makarov, Issue V, 1957.

87. Kel'zon, A. S. Homing as a Problem of Technical Cybernetics. Reports of Academy of Sciences SSSR, Vol. 116, Issue 6, 1957.

88. Kel'zon, A. S., and Grigor'yeva, O. V. Proportional Navigation as a Problem of Cybernetics. Reports of

Academy of Sciences SSSR, Vol. 121, Issue 3, 1958.

89. Kol'zon, A. S. A New Problem from the Area of Navigation. Works of LONTOVT, Issue 4, 1958.

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This publication was prepared under contract to the
UNITED STATES JOINT PUBLICATIONS RESEARCH SERVICE,
a federal government organization established
to service the translation and research needs
of the various government departments.